

# Chapter 6

## Exponential Smoothing and Adaptive Forecasting

New ways are continually being sought to forecast sales. One of the most exciting predicting notions which evolved out of World War II research is "exponential smoothing." This mathematical technique smooths data in a time series by a weighted moving average. The weights are a geometric progression with smallest weights assigned to oldest observations. When extrapolated with latest weights, the smoothed sales data become forecasts.

### 6.1 Basic Principle of Exponential Smoothing

Let us begin with the idea of a simple geometric progression:

$$1, (1-a), (1-a)^2, (1-a)^3, \dots, (1-a)^{n-1} \quad (6.1)$$

In this sequence, when the  $a$  value is between zero and one, each term becomes progressively smaller. For example, when  $a = \frac{1}{2}$ , the sequence becomes:

$$1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \dots, (\frac{1}{2})^{n-1} \quad (6.2)$$

If we apply these terms as weights to the successive observations of a sales series, a weighted moving average is computed as:

$$S_t(X) = \frac{1X_t + (1-a)X_{t-1} + (1-a)^2X_{t-2} + \dots + (1-a)^{n-1}X_{t-(n-1)}}{1 + (1-a) + (1-a)^2 + \dots + (1-a)^{n-1}} \quad (6.3)$$

where,

$X_t$  = value of sales in time period  $t$ .

$X_{t-i}$  = value of sales in a previous time period,  $t-i$  ( $i = 1, 2, 3, \dots$ ).

$S_t(X)$  = smoothed (moving average) value of sales in time period  $t$ .

$a$  = smoothing constant ( $0 < a < 1$ ).

Notice that the most recent data receive the largest weights of the geometric progression in Equation 6.1 and that the smoothed average changes as new data for  $X_t$  become available.

The operation of computing successive smoothed values has interesting algebraic properties<sup>1</sup> which yield a simplified form of Equation 6.3, such that only the old average is retained. The new average is estimated as:

$$S_t(X) = aX_t + (1-a)S_{t-1}(X) \quad (6.4)$$

where  $S_{t-1}(X)$  = smoothed value of sales in time period just prior to period  $t$ .

### 6.2 Example of Exponential Smoothing

Exponential smoothing is illustrated in Table 6.1 using data on "Beer Taxable Withdrawals." Withdrawals mean shipments out of the warehouse where the beer has been aging. This withdrawal defines a tax obligation. Sales for consumption normally follow withdrawals by a few weeks. Eight quarterly time periods appear in column 1, although months or some other interval might also be used. Actual unit withdrawals during 1971 and 1972 appear in column 2.

If we presume knowledge of actual withdrawals in the first quarter of 1971 (1-1971) of 266 (ten thousand) barrels, we may use this current information as well as the earlier forecast to predict withdrawals for quarter 2 (2-1972). Our first task is to determine the magnitude of importance we will give the actual withdrawals in 1-1971 versus that for the previous forecast. While we examine this issue in detail in the next section, let us for purposes of this discussion set  $a = 0.2$  (weight given to present actual withdrawals) and  $(1-a) = 0.8$  (complementary weight assigned to the old forecast). At the close of 1-1972 a forecast of 296.4 is forecast for 2-1972 as follows:

$$\begin{aligned} S_2(X) &= aX_2 + (1-a)S_{2-1}(X) \\ &= (0.2)(266) + (0.8)(304.0) \\ &= 53.2 + 243.2 \\ &= 296.4 \end{aligned} \quad (6.4)$$

These computations appear in Table 6.1, columns 3, 4, and 5.

Table 6.1

Exponential Smoothing Forecasts of U. S. Beer Taxable Withdrawals,  $\alpha = 0.2$ 

	(1)	(2)	(3)	(4)	(3) + (4) = (5)	(6)
Quarter and Year	Period t	Actual Withdrawals, $X_t$	Weighted Withdrawals, $\alpha X_t$	Weighted Previous Withdrawals Forecast,** $(1-\alpha)S_{t-1}(X)$	Current Withdrawals Forecasts,*** $\alpha X_t + (1-\alpha)S_{t-1}(X)$	Percent by which Forecast is high (+) or low (-). Col.(5) of previous row - Col. (2) of new row x 100 $\div$ Col. (2) of new row
Ten Thousand Barrels					Percent	
					(304.0)*	
1- 1971	1	266	53.2	243.2	296.4	+ 14.3
2-	2	311	66.2	237.1	303.3	- 10.5
3-	3	372	74.4	242.6	317.0	- 18.5
4-	4	305	61.0	253.6	314.6	+ 3.9
1- 1972	5	277	55.4	251.7	307.1	+ 13.6
2-	6	352	70.4	245.7	316.1	- 12.8
3-	7	382	76.4	252.9	329.3	- 17.3
4-	8	314	62.8	263.4	326.2	4.9
Totals		2,599	519.8	1,990.2	2,510.0	95.8****
Averages		324.9	65.0	248.8	313.8	12.0****

Notes on calculating procedure:

\*The initial forecast, 304.0 is the average of the four quarters for 1970.

\*\*The previous withdrawals forecast is based on the immediately preceding quarter; i.e., for first quarter 1971 243.2 is calculated as  $304.0 \times 0.8$ .

\*\*\*For each row the current withdrawals forecast is for the immediately following quarter; i.e.

296.4 ten thousand barrels is the forecast for 1971 quarter 2.

\*\*\*\*In summing column 6 signs are ignored, hence, the mean "absolute" percent difference equals 12.0.

Column 6 reveals one criterion by which forecast accuracy can be assessed. Percentage error or difference for the forecast for 1-1971 is calculated by:

$$\begin{aligned}\text{Percentage Difference} &= \frac{\text{Forecast} - \text{Actual}}{\text{Actual}} (100) \quad (6.5) \\ &= \frac{304.0 - 266.0}{266.0} \times 100 \\ &= +14.3\%\end{aligned}$$

That is, our forecast was high by 14.3% of actual withdrawals in 1-1971. The average percent difference at the bottom of column 6 is calculated by ignoring the signs of the percentage errors.

### 6.3 Selecting the Smoothing Constant, $\alpha$

The value for  $\alpha$  largely governs the amount of forecasting error incurred. Specifically, the smoothing constant governs the responsiveness of exponential smoothing to changes in actual sales. When  $\alpha$  is close to zero the smoothing process is relatively insensitive to erratic changes in the data. However, this insensitivity also causes its failure to adjust rapidly to actual trend changes. On the other hand, a large  $\alpha$  value enables the smoothing process to rapidly reflect changes in trend. Unfortunately, this sensitivity also fails to smooth over random data fluctuations. Figure 6.1 shows graphically these effects that changes in  $\alpha$  have on triple exponentially smoothed data compared with the original series for Auto Dealer Retail Sales.<sup>2</sup>

The objective is to choose the value for  $\alpha$  which produces the smallest forecast error. The method is an iterative trial-and-error procedure of comparing mean absolute percent differences for various  $\alpha$  values. For example, Figure 6.2 illustrates the effect of the smoothing constant on forecasting error for Auto Dealer Retail Sales.

In this example,  $\alpha = 0.3$  results in the smallest error among the  $\alpha$  values 0.1, 0.3, 0.5, 0.7, and 0.9.

### 6.4 Double and Triple Exponential Smoothing and Adaptive Forecasting

Exponential smoothing discussed thus far is termed "simple." It assumes the underlying data have a level trend with random fluctuations around this trend. Exponential smoothing of higher order can cope with a non-zero sloped trend.

Since simple exponential smoothing typically lags behind any trend in the original data, we will try to correct for the magnitude and direction of the sales trend. One procedure called *double* exponential smoothing utilizes the differences between successive forecasts as a means of computing the average trend. This trend estimate is then weighted by  $\alpha$ , and the result is added to each current forecast to adjust the forecast for trend. Clark and Schkade<sup>3</sup> outline the three steps to double exponential smoothing as follows:

1. Change in average = new average - old average or

$$C_t = S_t(X) - S_{t-1}(X) \quad (6.6)$$

where  $C_t$  = change in the smoothed value between time period  $t$  and the previous time period.

2. New trend =  $\alpha(\text{change in average}) + (1 - \alpha)$  old trend or

$$T_t = \alpha C_t + (1 - \alpha) T_{t-1} \quad (6.7)$$

where  $T$  = trend.

3. Expected sales (forecast) = new average +  $(1 - \alpha)/\alpha$  (new trend) or

$$S_{t+1} = S_t(X) + \frac{1 - \alpha}{\alpha} T_t \quad (6.8)$$

where  $S_{t+1}$  = expected sales or the forecast for the next time period.

A further extension of the previous idea is *triple* exponential smoothing. If the sales trend is not linear, the case may call for third order smoothing with a second-degree polynomial. The data are assumed to follow a parabola of the form  $A + Bt + Ct^2/2$ , where  $A$ ,  $B$ , and  $C$  are parameters which change after each new data point is added and  $t$  is time.

Double and triple, as well as simple, exponential smoothing are exceedingly tedious when calculated by hand. In fact, exponential smoothing probably is not worthwhile without computer assistance. Most computer libraries today include exponential smoothing programs and, therefore, detailed manual computations have been omitted here.<sup>4</sup>

Extensions of exponential smoothing to include computer selection of parameters for trend equations used to forecast the future have been published under the title of *adaptive forecasting*. For example, Figure 6.1 is derived using a computerized triple exponential smoothing program in which initial values for the parameters  $A$ ,  $B$ , and  $C$  are calculated using the first three data points. These beginning estimates are then refined with each iteration of the exponential smoothing process. Alternatively, for this particular program the user is given the option to input the initial parameter estimates manually.

A still further extension of exponential smoothing concepts is illustrated by Benton in an example where the level of the trend is corrected each period by a fraction of the previous error and where the slope of the trend is also corrected each period by a fraction of the previous error in the slope. These continuous correction concepts are then combined with a measure of significant deviation from average error to yield a "tracking signal." The method is described in more detail in Chapter 16 in the monitoring discussion.

### 6.5 Adjustment for Seasonal Influences

The nature of seasonal influences relative to exponential smoothing is demonstrated by several examples. Figure 6.3 illustrates triple exponential smoothing applied to Safeway original quarterly sales. In Figure 6.4 triple exponential smoothing is again applied, but this time to seasonally adjusted Safeway data. A comparison of these two figures shows that if time series observations are not seasonally

Figure 6.1

Auto Dealer Retail Sales

Annual Forecasts by Exponential Smoothing

Sales: Billion Dollars

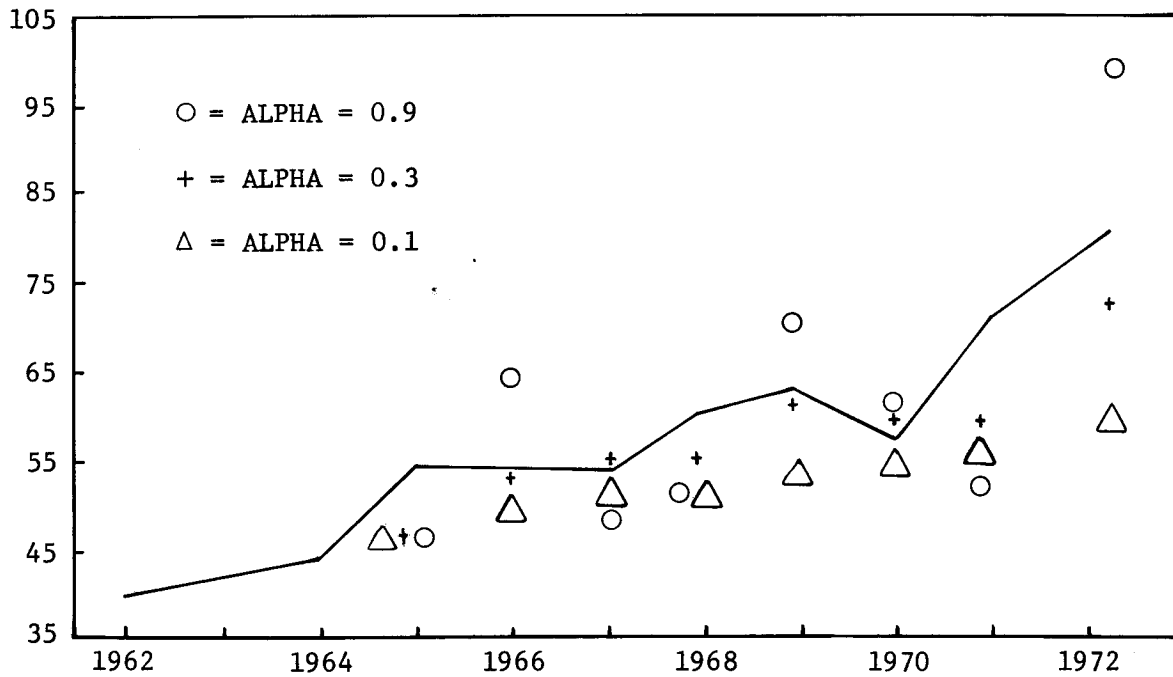


Figure 6.2

Auto Dealer Retail Sales

Effect of Smoothing Factor on Forecasting Error

Error: Mean Absolute Percent Deviation

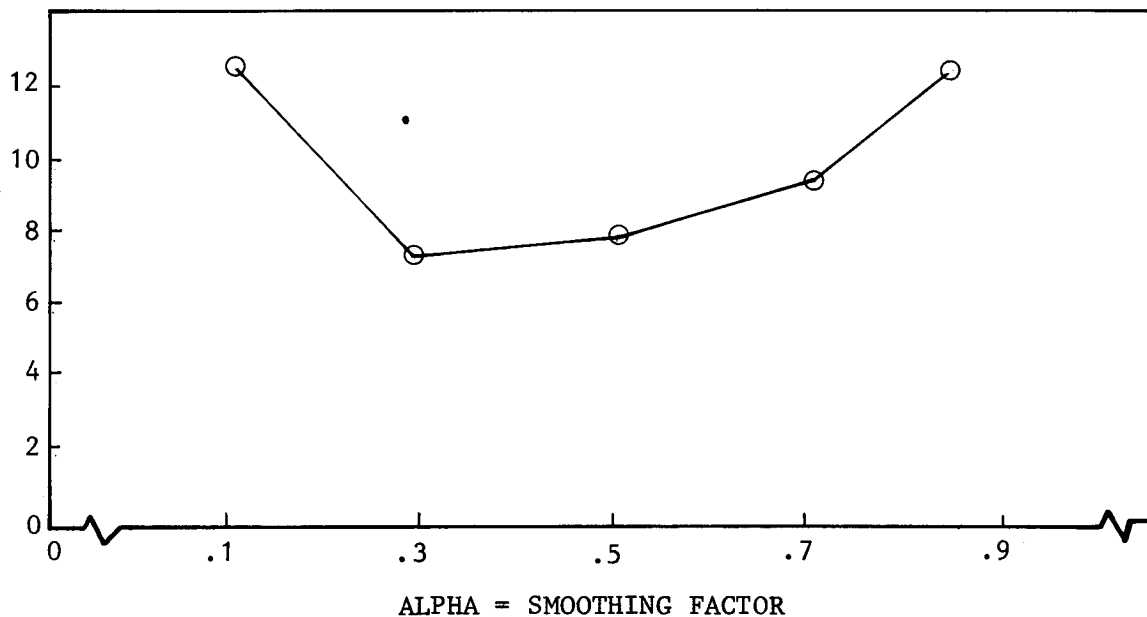


Figure 6.3  
Safeway Stores Incorporated

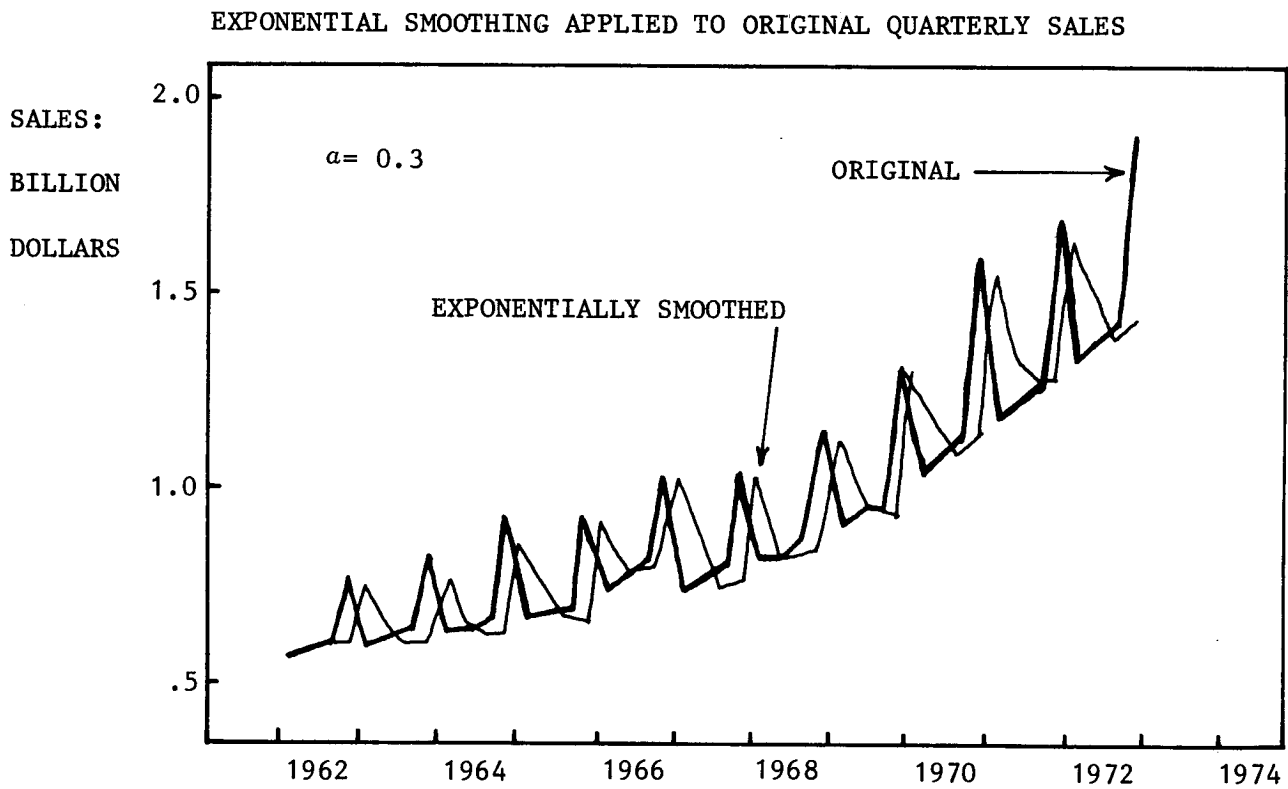


Figure 6.4  
Safeway Stores Incorporated  
EXPONENTIAL SMOOTHING APPLIED TO SEASONALLY ADJUSTED  
QUARTERLY SALES

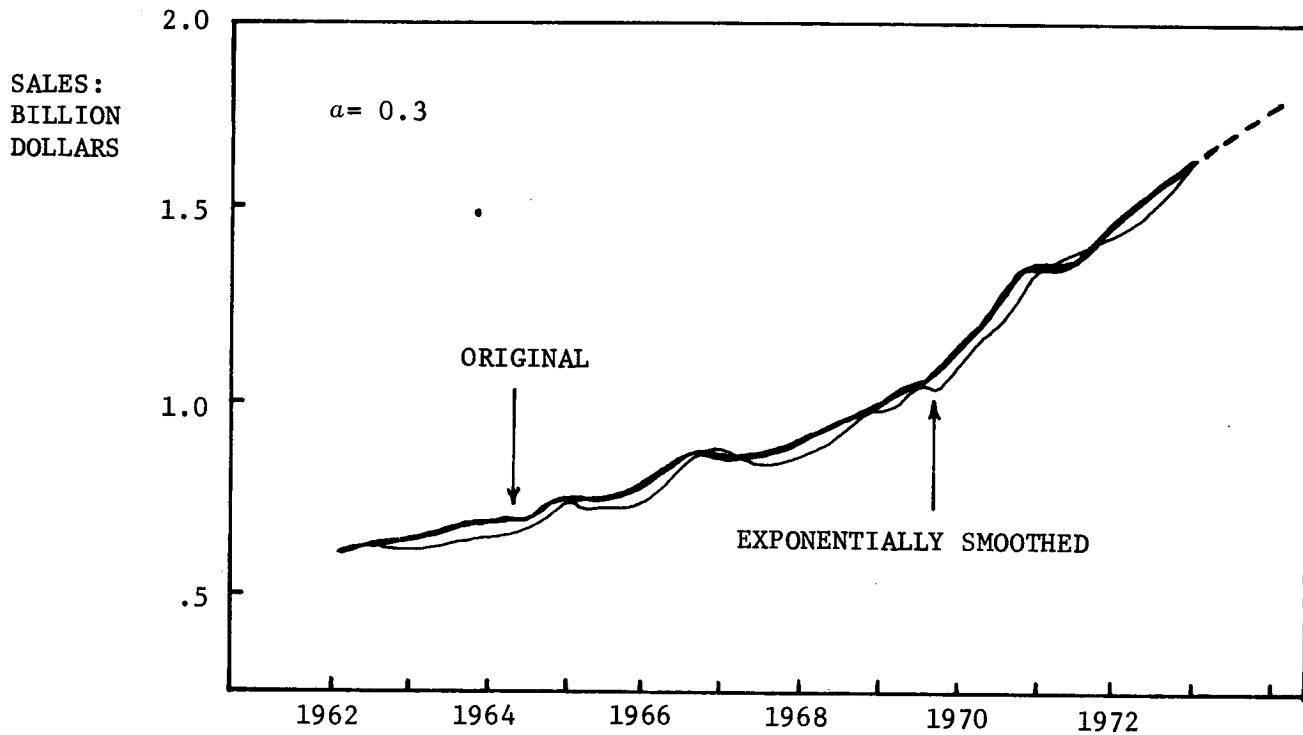
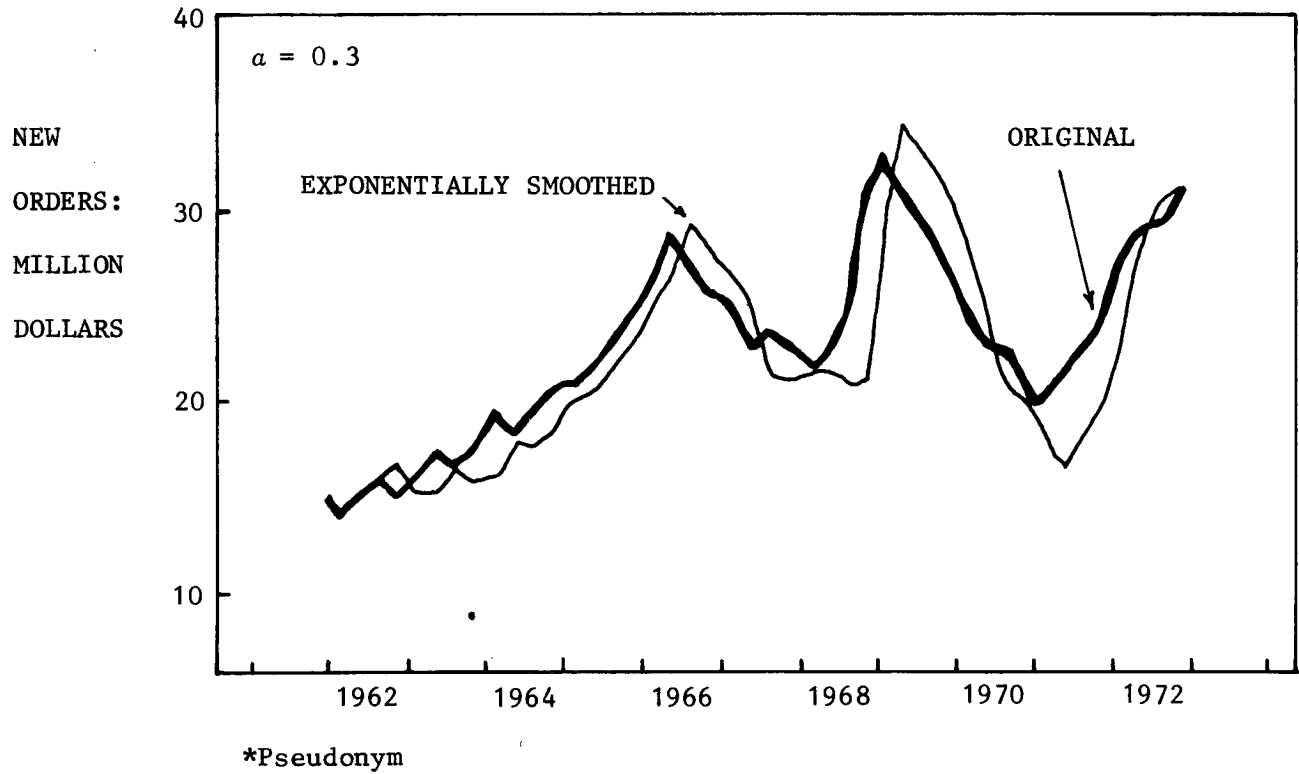


Figure 6.5  
Process Control Company\*

EXPONENTIAL SMOOTHING APPLIED TO SEASONALLY ADJUSTED  
QUARTERLY SALES



adjusted (Figure 6.3), exponentially smoothed data will typically lag the seasonal pattern to a greater extent than if seasonal adjustment were first applied (Figure 6.4). The obvious exception would be when seasonal influences are small.

## 6.6 Exponential Smoothing of Cyclical Data

In Figure 6.5, Process Control Company data demonstrate the shortcoming that exponential smoothing will always lag business cycle turning points. The implication, then, is that exponential smoothing is inappropriate to use on data which have significant business cycle components.

## 6.7 Advantages and Disadvantages of Exponential Smoothing

The advantages of forecasting sales by exponential smoothing include the following:

1. Required data are minimum.
2. Calculations are readily computerized.
3. Little analytical skill is needed.
4. Method is cheap and rapid.
5. Method is objective.
6. Method is easily understood.

The disadvantages of forecasting by exponential smoothing are:

1. Method misses turning points.
2. Method assumes continuity of the future with the past.
3. Method gives no response to company marketing strategy.
4. Method makes no explicit use of prospective market developments.

Since exponential smoothing is both inexpensive and fast (when a computer is used), it is especially appropriate when a large number of series must be forecast. Many companies sell thousands of different types of products, and it could be prohibitively expensive to employ forecasting methods to predict sales for each item by the sophisticated forecasting methods discussed later.

## Footnotes

1. See Clifford H. Springer et al., *Advanced Methods and Models* (Homewood, Illinois, Richard D. Irwin, Inc., 1965), p. 95.
2. See Section 6.4 for further explanation regarding Figure 6.1.
3. Charles T. Clark and Lawrence L. Schkade, *Statistical Methods for Business Decisions* (Dallas, Texas, South-Western Publishing Company, 1969), pp. 705-706.
4. The reader who is interested in pursuing the mathematics of "higher order" exponential smoothing should refer to: Robert G. Brown, *Smoothing, Forecasting and Prediction of Discrete Time Series* (Englewood Cliffs, New Jersey, Prentice-Hall, Inc., 1963), pp. 123-144.

## Bibliography

- Battersby, Albert. *Sales Forecasting*. London, England: Cassell & Company, Ltd., 1968, ch. 3.
- Benton, William D. *Forecasting for Management*. Reading, Massachusetts: Addison-Wesley Publishing Company, 1972, ch. 4.
- Bolt, Gordon J. *Market and Sales Forecasting—A Total Approach*. New York: John Wiley and Sons, Inc., 1972, ch. 7.
- Brown, Robert G. *Smoothing, Forecasting, and Prediction of Discrete Time Series*. Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1963.
- Chisholm, Roger K. and Gilbert R. Whitaker, Jr. *Forecasting Methods for Business Decisions*. Dallas, Texas: South-Western Publishing Company, 1969, ch. 20.
- Clark, Charles T. and Lawrence L. Schkade. *Statistical Methods for Business Decisions*. Dallas, Texas: South-Western Publishing Company, 1969, ch. 20.
- Enrick, Norbert L. *Market and Sales Forecasting*. San Francisco: Chandler Publishing Company, 1969, ch. 2.
- Groff, Gene K. and John F. Muth. *Operations Management: Analysis for Decisions*. New York: Richard D. Irwin, Inc., 1972, ch. 11.
- Lippitt, Vernon G. *Statistical Sales Forecasting*. New York: Financial Executives Research Foundation, 1969, Appendix B.
- Nelson, Charles. *Applied Time Series Analysis*. San Francisco: Holden-Day, Inc., 1973.
- Neter, John et al. *Fundamental Statistics for Business Economics*. Boston: Allyn and Bacon, Inc., 1973, ch. 32.
- Springer, Clifford H. et al. *Advanced Methods and Models, Vol. II of the Mathematics for Management Series*. Homewood, Illinois: Richard D. Irwin, Inc., 1965, ch. 4.